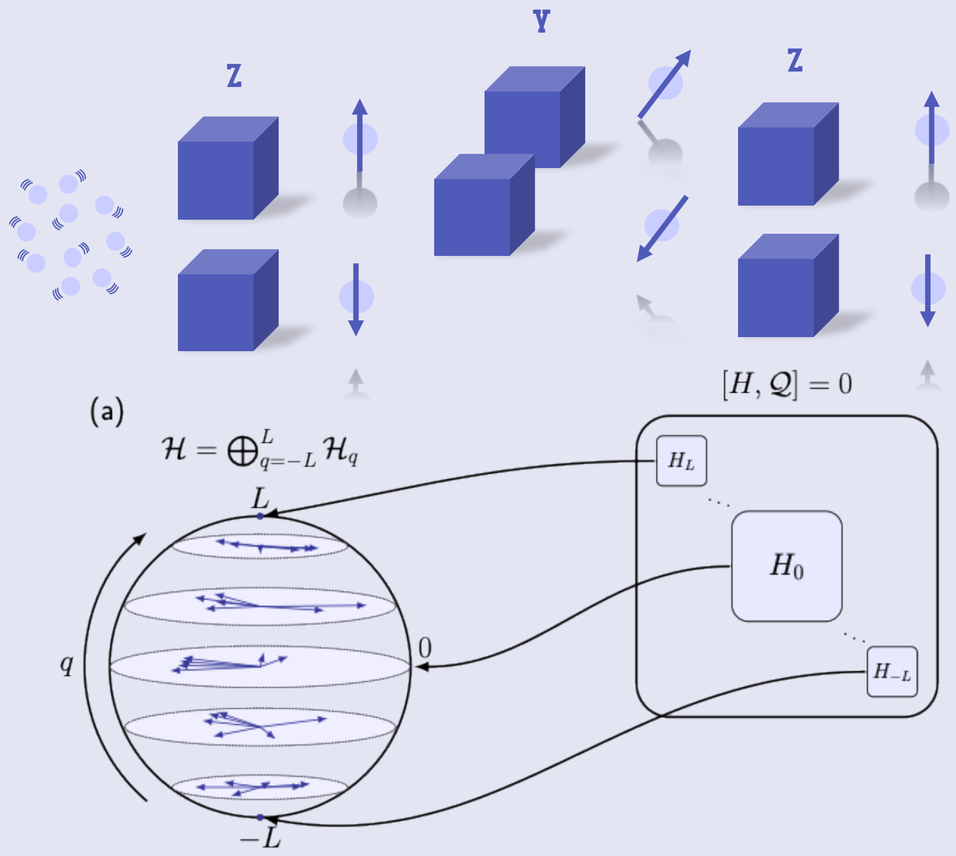
MAGIC AND RANDOMNESS: ABELIAN SYMMETRIES IN CHAOTIC MANY-BODY QUANTUM SYSTEMS



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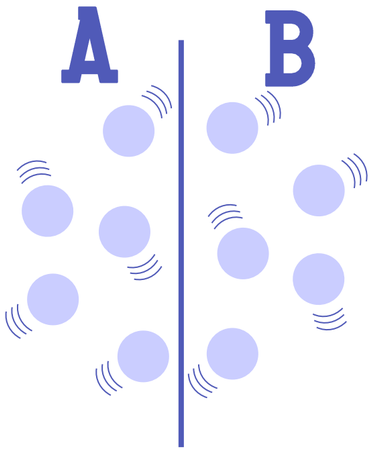
QUANTUM INFORMATION THEORY LAB

QUANTUM RESOURCES FOR QUANTUM COMPUTING

ENTANGLEMENT

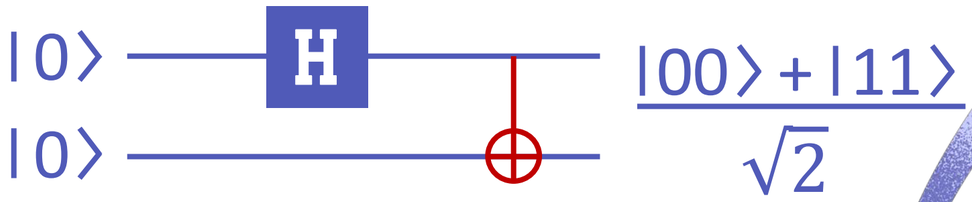
MEANING

Non-classical correlations across subsystems



FREE OPERATIONS

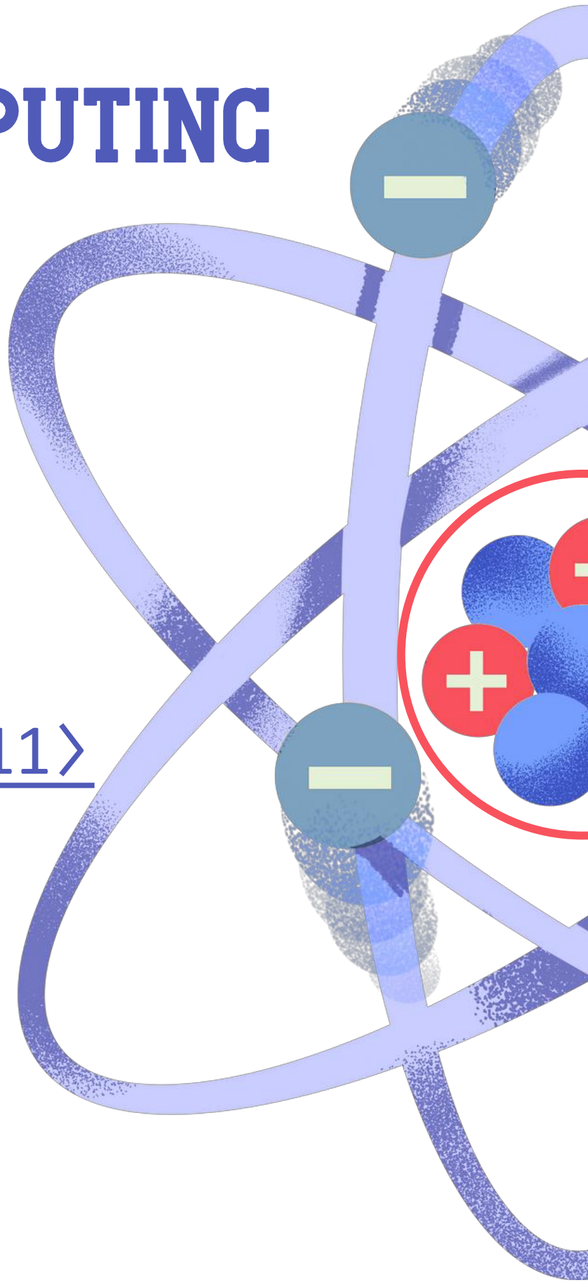
Local Operations and Classical communications



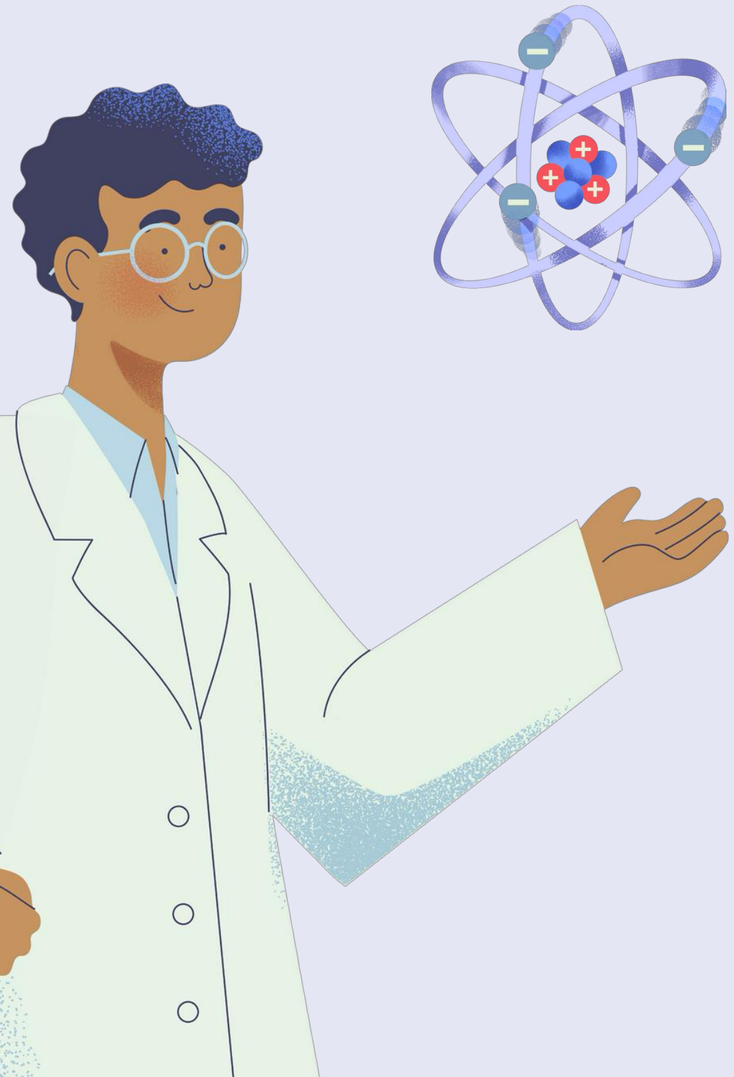
EXPERIMENTALLY MEASURABLE QUANTITY

Subsystem purity

$$\mathrm{E}_{lin}(\psi) = 1 - Tr(\psi_A^2)$$



R. Horodecki, et al. "Quantum entanglement." Reviews of modern physics 81.2 (2009): 865-942.



IS ENTANGLEMENT ENOUGH?

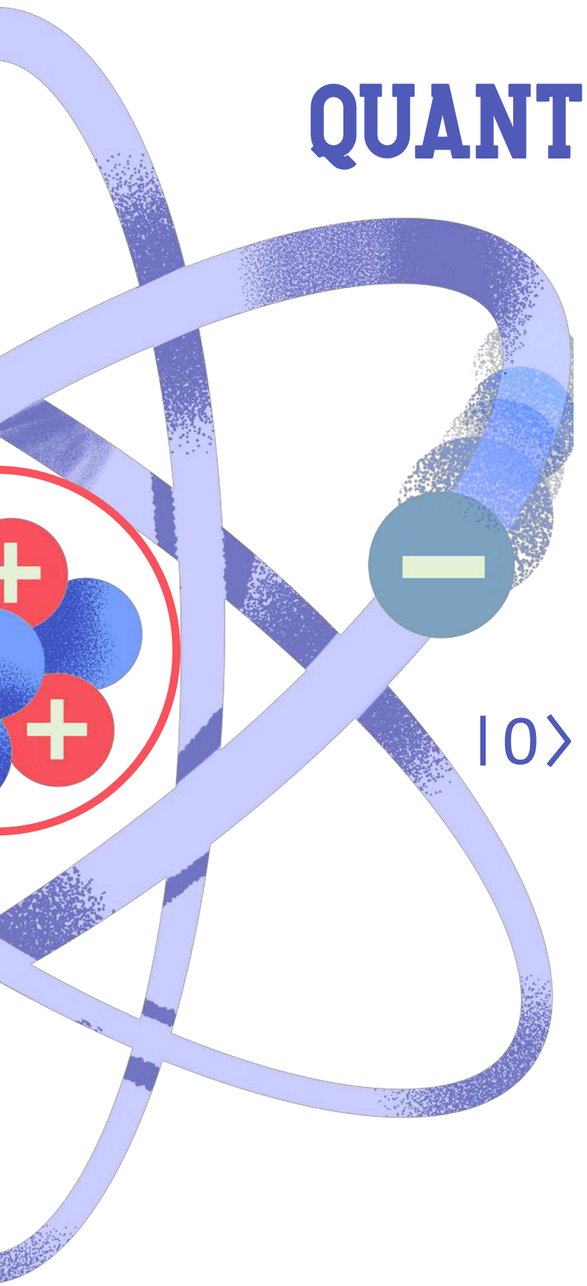
What happens when we have only a single particle? 1

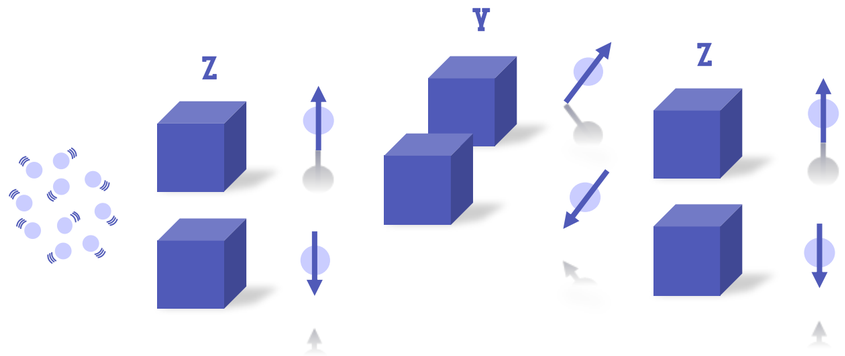
Theorem: A quantum circuit using only the following elements can be simulated efficiently on a classical computer: 2

- **Preparation of qubits in *computational-basis states***
- ***Clifford gates***
- ***Measurements in the computational basis***

Aaronson, S., & Gottesman, D. (2004). Improved simulation of stabilizer circuits. Physical Review A, 70(5), 052328.

QUANTUM RESOURCES FOR QUANTUM COMPUTING





$$|0\rangle \text{ — } \boxed{H} \text{ — } \boxed{T} \text{ — } \frac{|0\rangle + e^{i\frac{\pi}{4}}|1\rangle}{\sqrt{2}}$$

$$M_{lin}(\psi) = 1 - \frac{1}{d}\sum_{P} Tr^4(P\psi)$$

MEANING

Partial incompatibility

FREE OPERATIONS

Stabilizer operations

EXPERIMENTALLY MEASURABLE QUANTITY

Stabilizer purity

NON-STABILIZERNESS

H. Zhu, R. Kueng, M. Grassl& D. Gross. “The Clifford group fails gracefully to be a unitary 4-design.” arXiv:1609.08172 (2016).

L. Bittel, and L. Leone. "Operational interpretation of the Stabilizer Entropy." arXiv:2507.22883 (2025).

NON-STABILIZERNESS

Stabilizer/Little Group:

$$G_\phi = \{g \in G \mid g \cdot \phi = \phi\} \subset G$$

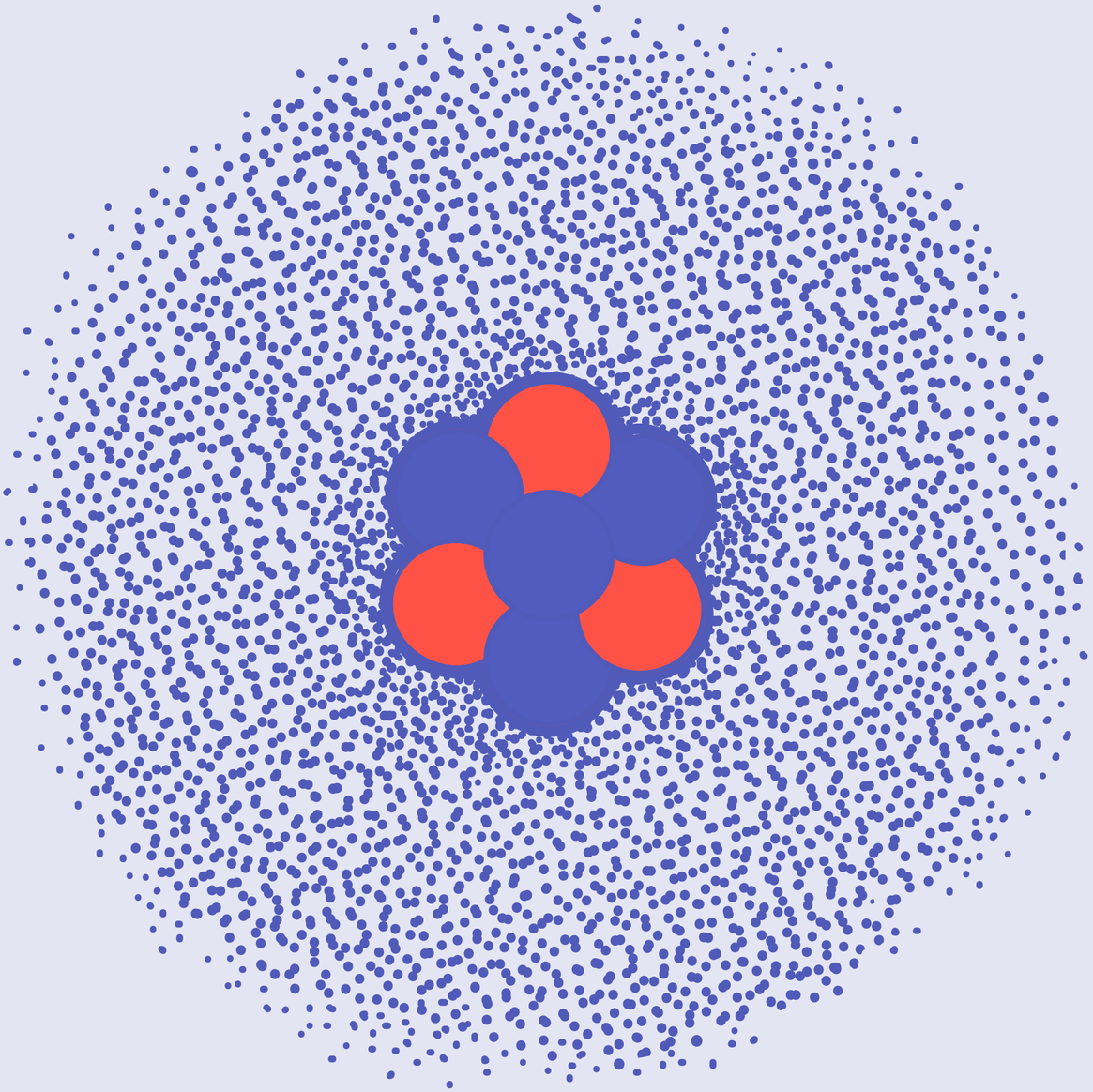
Pauli Group:

$$P_n = \{\pm 1, \pm i\} \times \{\mathbb{I}, X, Y, Z\}^{\otimes n}$$

Pure stabilizer states

$$\mathrm{PSTAB} = \{|\phi\rangle\langle\phi| = \frac{1}{|G_\phi|} \sum_{P \in G_\phi} P \mid G_\phi \subset P_n\}$$





WHAT IS THE UNBIASED DISTRIBUTION FOR QUANTUM STATES?

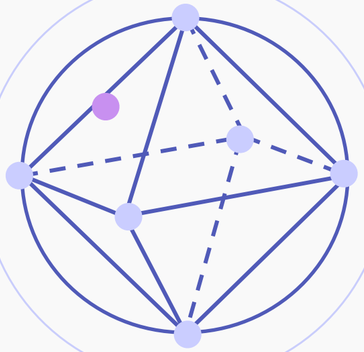
UNIFORM DISTRIBUTION

$$|\psi\rangle = U\,|0\rangle, U \sim Haar\big(U(d)\big)$$

$$\mathbb{E}[f(\psi)] = \int f(U|0\rangle)\,dU$$

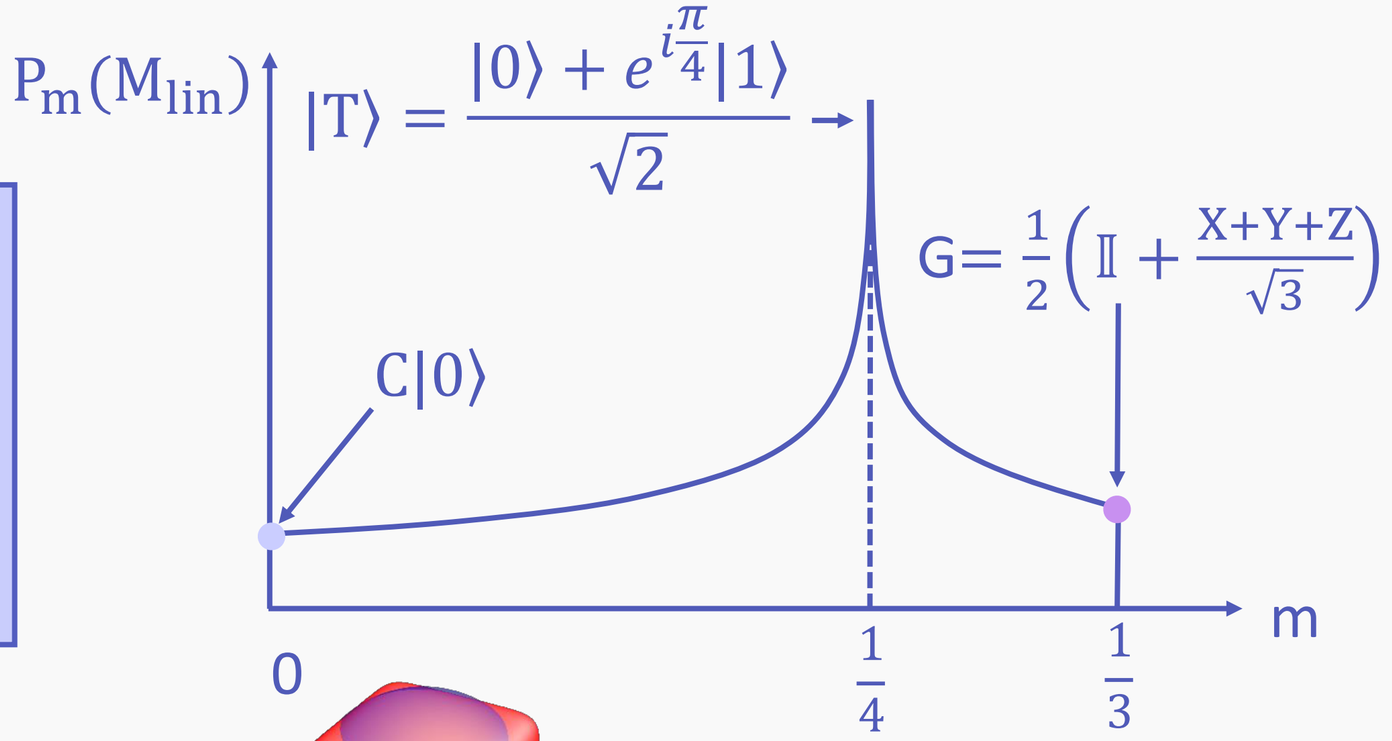
I. Bengtsson and K. Zyczkowski, Geometry of Quantum States: An Introduction to Quantum Entanglement. Cambridge University Press, 2006.

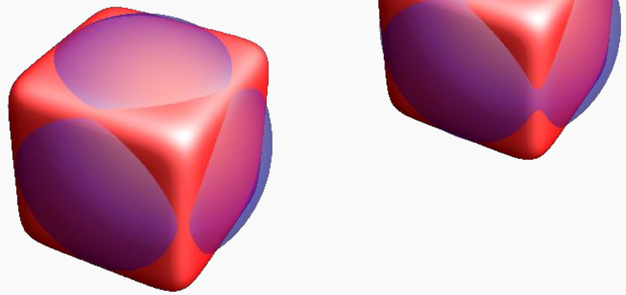
PROBABILITY DENSITY FUNCTION OF MAGIC



There are no divergences in Hilbert spaces $d > 2$ for the PDF of magic

Magic concentrates around its mean value for *high-dimensional Hilbert* spaces in the Lévy sense

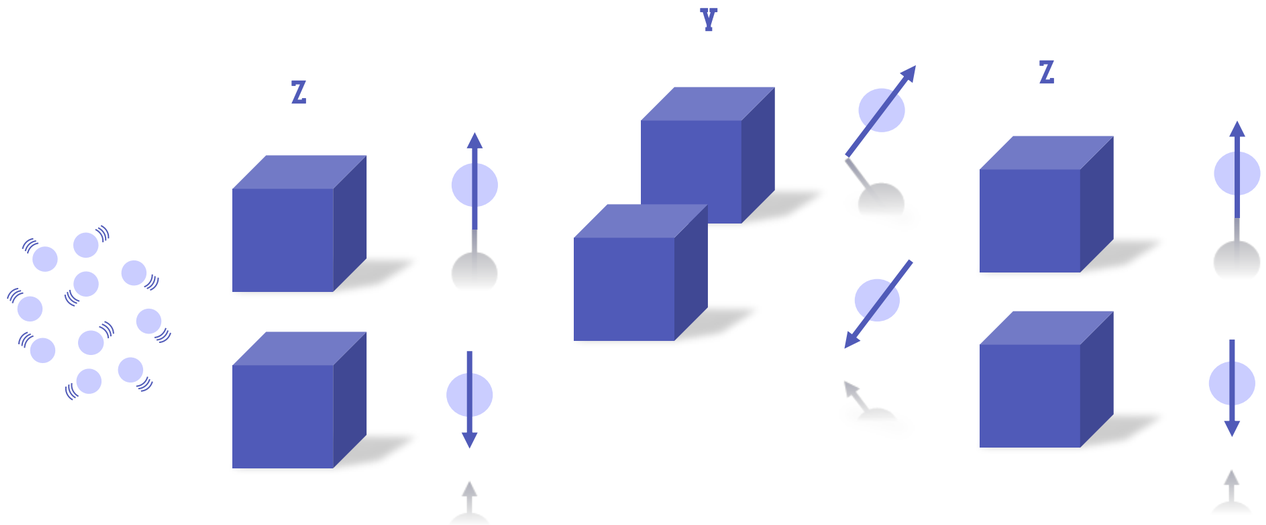




The PDF for one qubit exhibits logarithmic Van Hove singularity

D.I, L. Campos Venuti and A. Hamma. "Van Hove singularities in stabilizer entropy densities." J. Ph. A: Mat. and Th., 2026, doi: 10.1088/1751-8121/ae3cd6

PARTIAL INCOMPATIBILITY



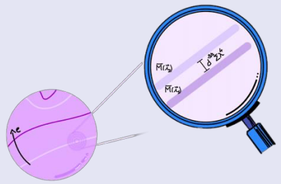
Partial incompatibility

$$\Gamma_\alpha(\psi) := \sum_{\mathrm{P}} \|[\psi, \mathrm{P}]\|_{2\alpha}^{2\alpha}$$

Relation with Non-stabilizerness

$$\Gamma_2(\psi) = 2d\big(d - 1 - \mathrm{M}_{\mathrm{lin}}(\psi)\big)$$

D.I, L. Campos Venuti and A. Hamma. "Van Hove singularities in stabilizer entropy densities." J. Ph. A: Mat. and Th., 2026, doi: 10.1088/1751-8121/ae3cd6

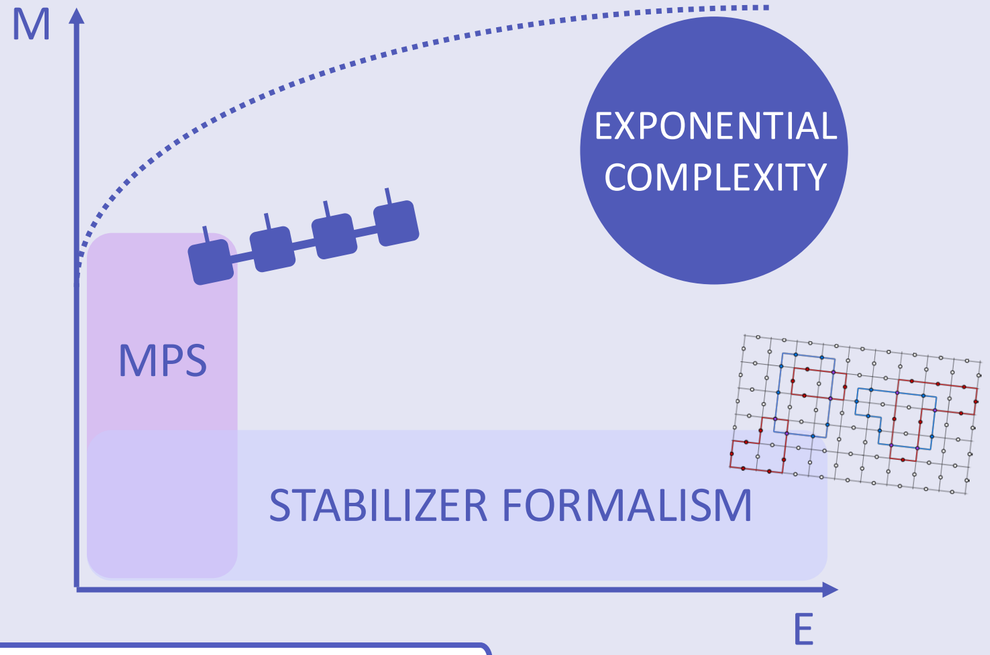


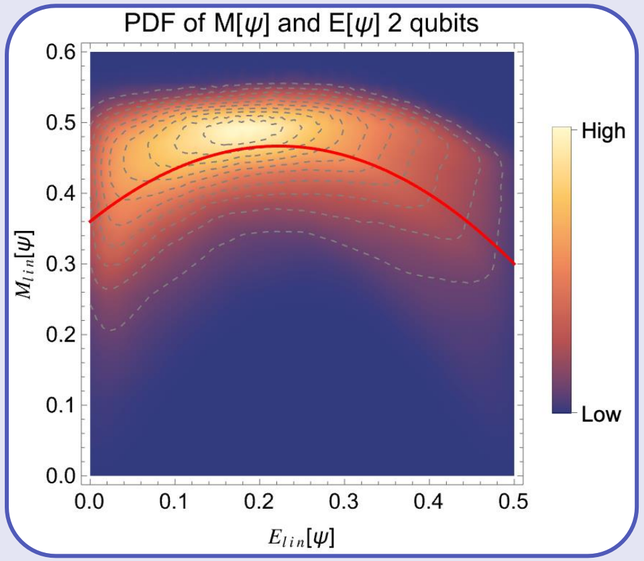
INTERPLAY BETWEEN MAGIC AND ENTANGLEMENT

1. Entanglement makes room for magic:

The separable state with maximal magic on a single qubit is much less resourceful than its average

$$\mathrm{M_{lin}}\left(\mathrm{G}^{\otimes \mathrm{n}}\right) = 1 - \left(\frac{2}{3}\right)^{\mathrm{n}} < \mathbb{E}[\mathrm{M_{lin}}(|\psi\rangle)]$$





2. RESULTS:

$$\mathbb{E}[\mathrm{M_{lin}}(\mathrm{U_A}\otimes \mathrm{U_B}|\psi\rangle)] = \alpha + \beta\ \mathrm{E}_{lin}(\psi) + \gamma\ \mathrm{E}_{lin}(\psi)^2 + \delta[\mathcal{F}(\psi) + (1 - \mathrm{E}_{lin}(\psi)^2)] + \mu \sum_i \lambda_i^4 ,$$

$$\mathrm{Cov}(\mathrm{E_{lin}}, \mathrm{M_{lin}}) = \mathbb{E}[\mathrm{M_{lin}}\ \mathrm{E_{lin}}] - \mathbb{E}[\mathrm{M_{lin}}]\mathbb{E}[\mathrm{E_{lin}}] = 0$$

3. TO REMEMBER:

$$\mathcal{F}(\psi) = Tr[\psi^3] - Tr^2[\psi^2]$$

$$\mathbb{E}_{\mathrm{C}}[\mathcal{F}(\mathrm{C}|\psi\rangle)] \propto \mathrm{M_{lin}}\,(|\psi\rangle)$$

D.I., et al. "Entanglement and Stabilizer entropies of random bipartite pure quantum states." Quantum, vol.9, p. 1797, July 2025

Tirrito, Emanuele, et al. "Quantifying nonstabilizerness through entanglement spectrum flatness." Physical Review A 109.4 (2024): L040401.

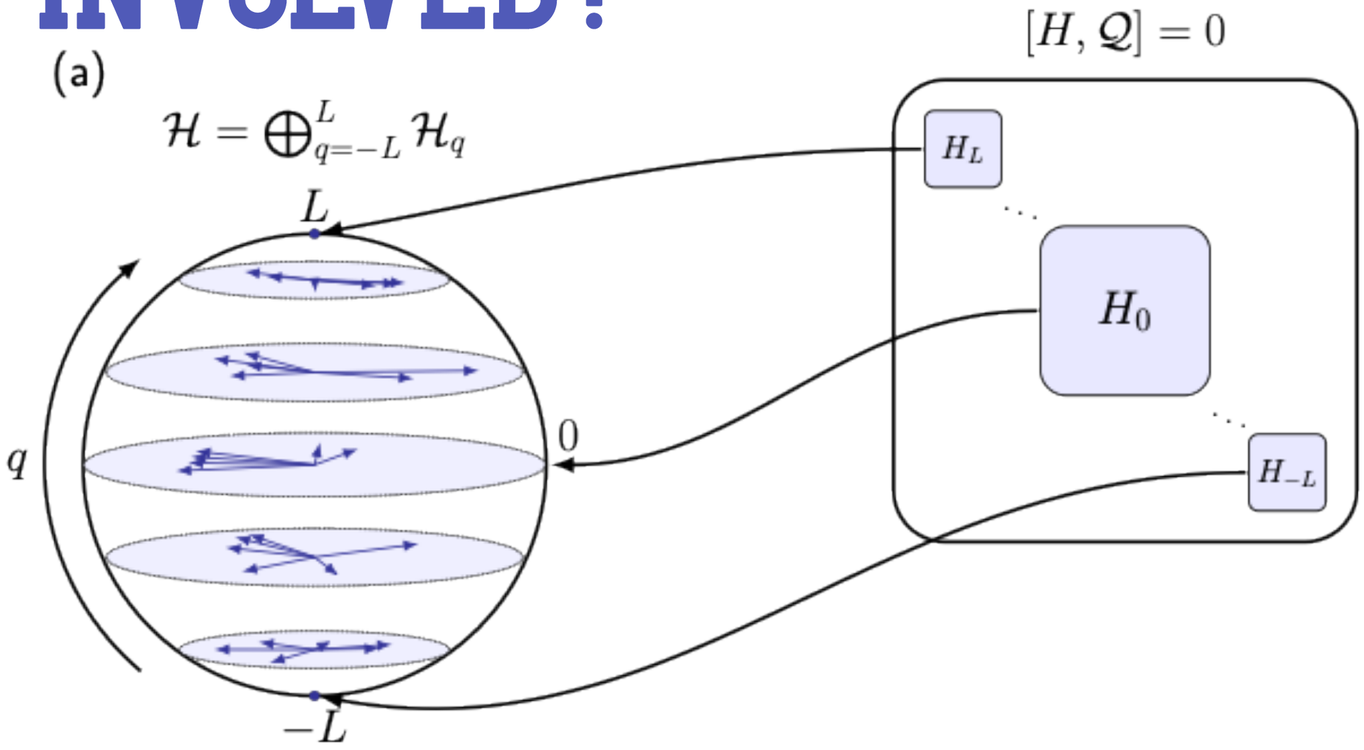
WHAT HAPPENS WHEN SYMMETRIES ARE INVOLVED?

$$\mathrm{H} = \sum_{\mathrm{n}} E_n |\psi_n\rangle\langle\psi_n|$$

Example of a U(1) charge

$$Q = \sum_{j=1}^{\mathrm{L}} Z_j \text{ and } [H, Q] = 0$$

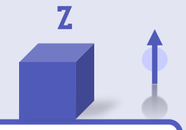
$$\mathcal{H} = \bigoplus_{\mathrm{q=-L}}^{\mathrm{L}} \mathcal{H}(\mathrm{q}) \ni |\psi_{\mathrm{q}}\rangle \equiv |\psi_n\rangle$$



$$\mathbb{E}_{\mathrm{U_q}}\left[\psi_{\mathrm{q}}^{\otimes \mathrm{k}}\right] \propto \Pi_{\mathrm{q}}^{\otimes \mathrm{k}}\, \mathbb{E}_{\mathrm{U}}\left[\psi^{\otimes \mathrm{k}}\right]$$

Bianchi, E., Hackl, L., Kieburg, M., Rigol, M. and Vidmar, L., 2022. Volume-law entanglement entropy of typical pure quantum states. PRX quantum, 3(3), p.030201.
Cepollaro, S., Chirco, G., Cuffaro, G., Esposito, G. and Hamma, A., 2024. Stabilizer entropy of quantum tetrahedra. Physical Review D, 109(12), p.126008.

AVERAGE & VARIANCE



Exact average:

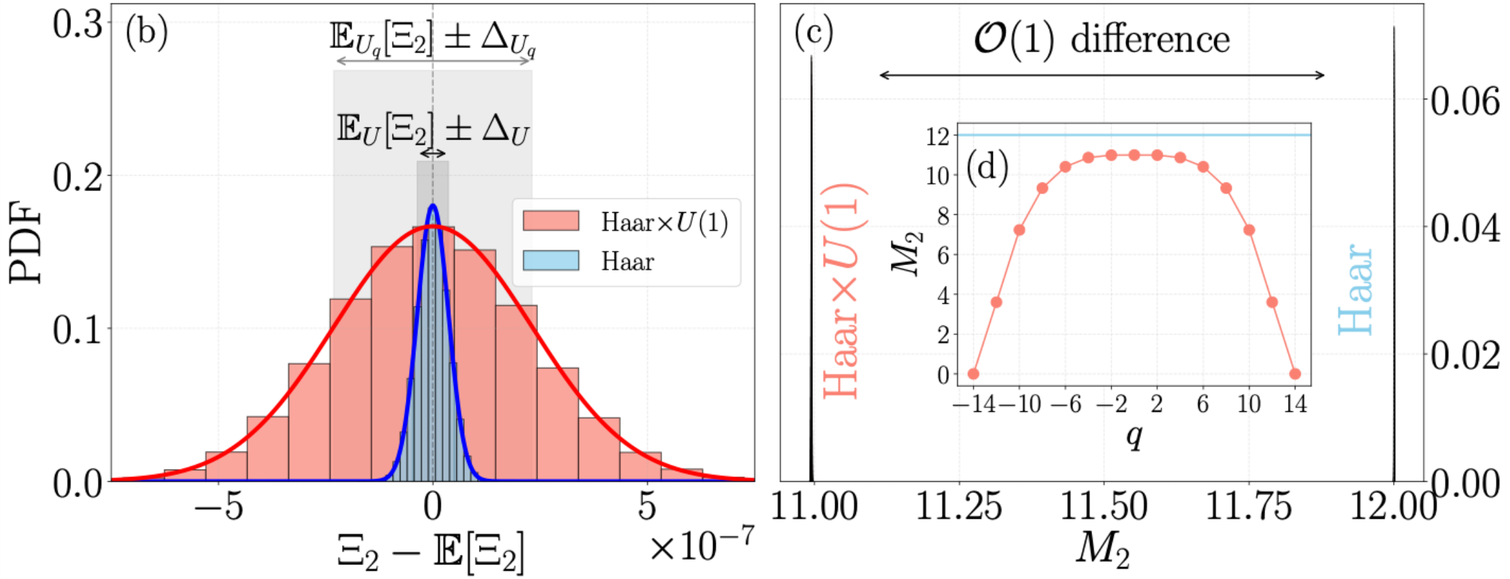
$$\mathbb{E}_{U_q}\left[\frac{1}{d}\sum_{P} Tr^4(P\psi_q)\right] = \frac{12\,d_q^2 + 8\,d_q + 2^{2-L}\sum_{k=0}^{L}\binom{L}{k}\left[\sum_{m=0}^{k}(-1)^m\binom{L-k}{\frac{L-q}{2}-m}\binom{k}{m}\right]^4}{4!\binom{d_q+3}{4}}$$

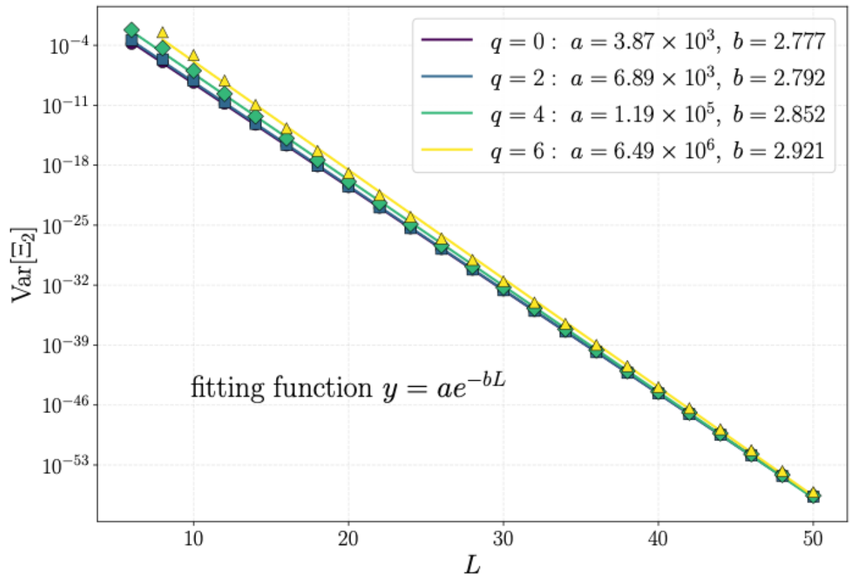
Lévy typicality

Asymptotic:

$$\lim_{L\to\infty} E_{U_s}\left[M_2(\psi_{U_s})\right] \simeq_{s\to 0} L - 3 + \frac{L-3}{\log 2}s^4 + O(s^6)$$

$$s = q/L$$



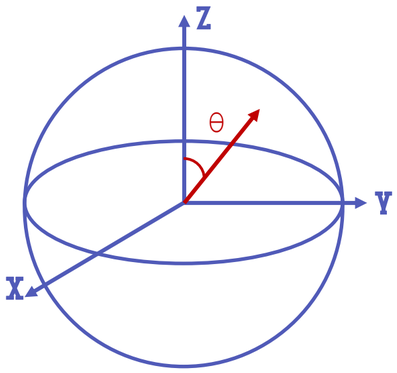


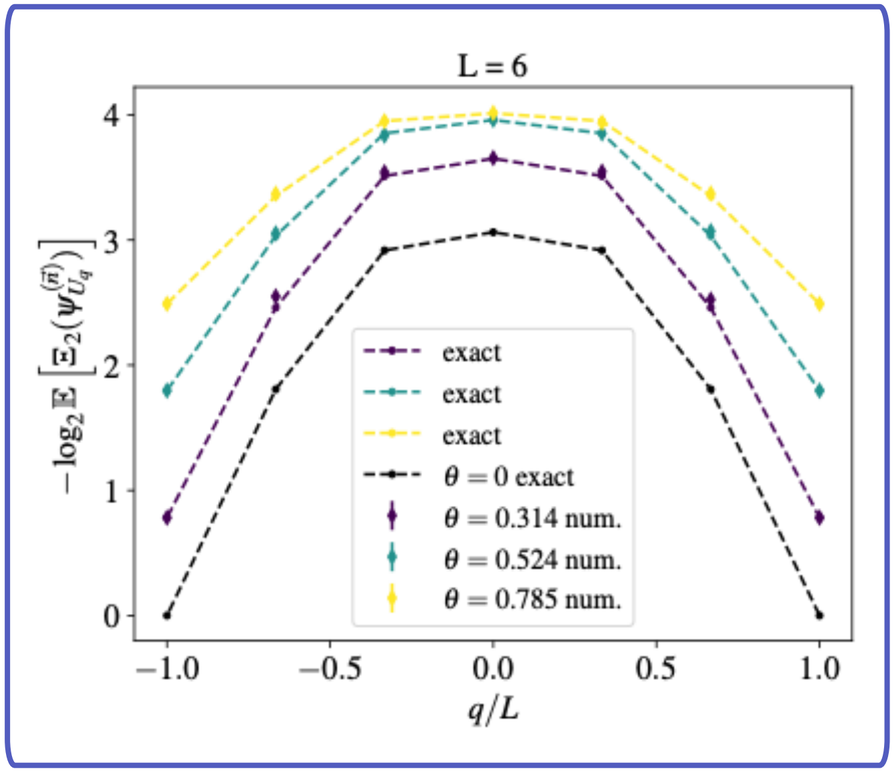
D.I. , A. Russotto, B. Jasser, J. Odavic and A. Hamma " Non-stabilizerness and U(1) symmetry in chaotic many-body quantum systems" soon on arxiv

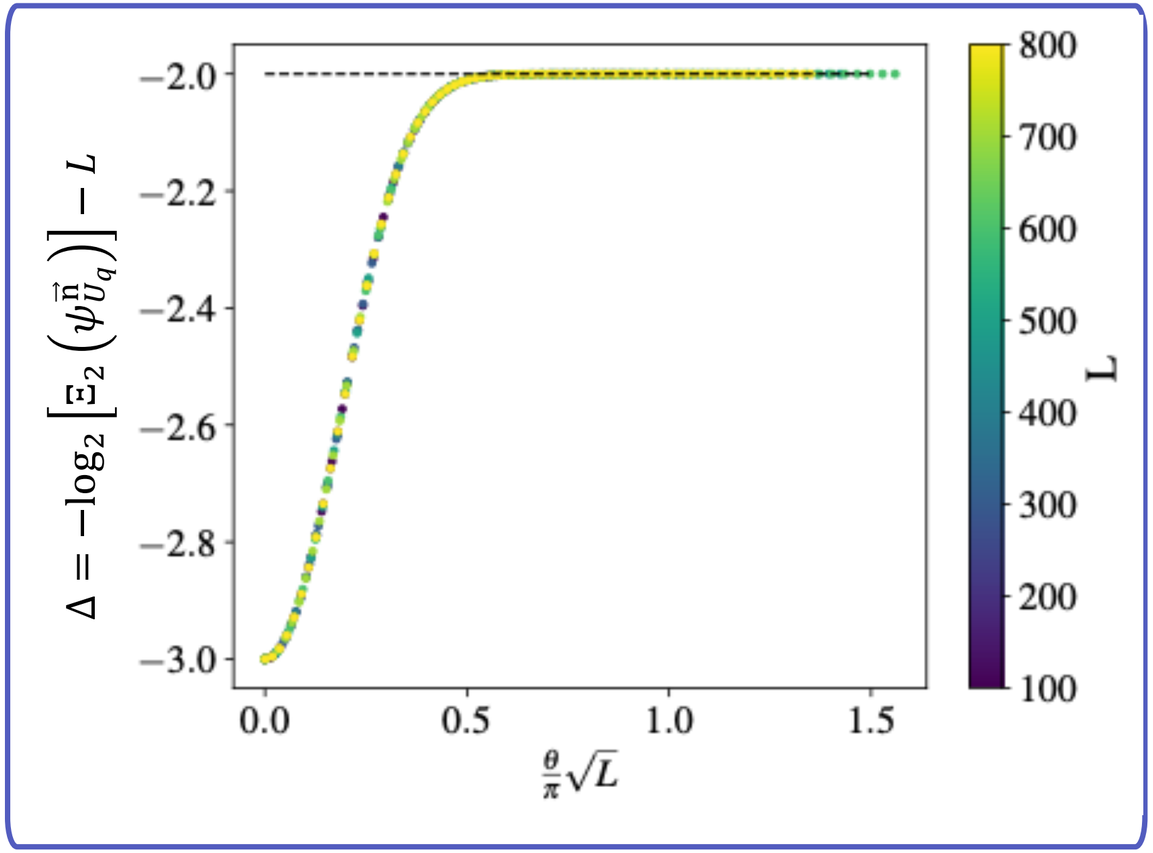
AVERAGE: GENERIC DIRECTION

Asymptotic:

$$\lim_{L\to\infty} E_{U_0}\left[M_2\left(\psi^{\vec{n}}_{U_0}\right)\right] = \begin{cases} L-3 & \vec{n} = \hat{x}, \hat{y}, \hat{z} \\ L-2 & \text{otherwise} \end{cases}$$



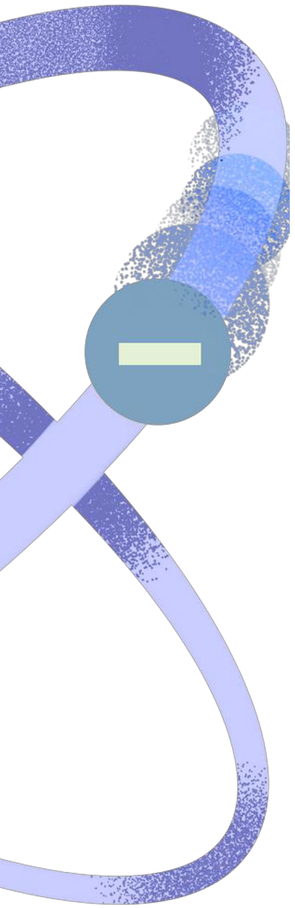


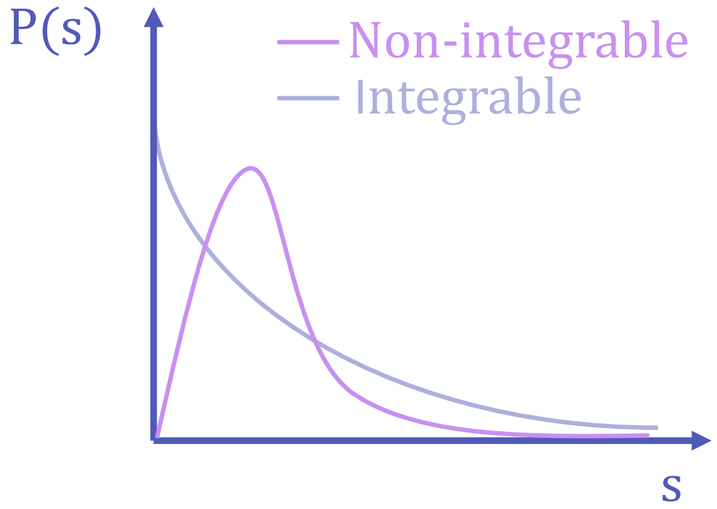


Different conserved charges parametrized by $\vec{n} = (\sin\theta, \cos\theta, 0)$

D.I. , A. Russotto, B. Jasser, J. Odavic and A. Hamma " Non-stabilizerness and U(1) symmetry in chaotic many-body quantum systems" soon on arxiv

QUANTUM MANY-BODY CHAOTIC HAMILTONIANS





HAMILTONIAN

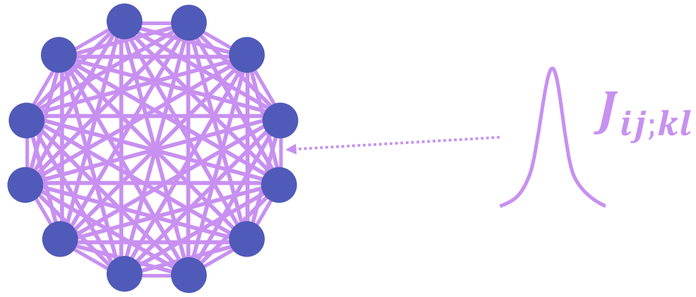
$$H = \sum_n E_n |\psi_n\rangle\langle\psi_n|$$

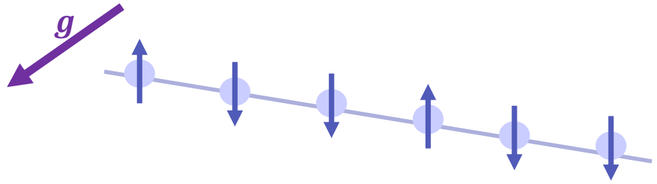
LEVEL SPACING

$$s_n = E_{n+1} - E_n$$

Sachdev-Ye-Kitaev model

$$H_{SYK} = \frac{1}{(2L)^{3/2}} \sum_{i,j,k,l=1}^{L} J_{ij;kl}\, c_i^\dagger c_j^\dagger c_k c_l$$



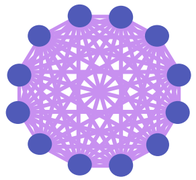


Transverse field Ising model

$$H_{TFIM} = \sum_{j=1}^{L} (Z_j Z_{j+1} + g\, X_j)$$

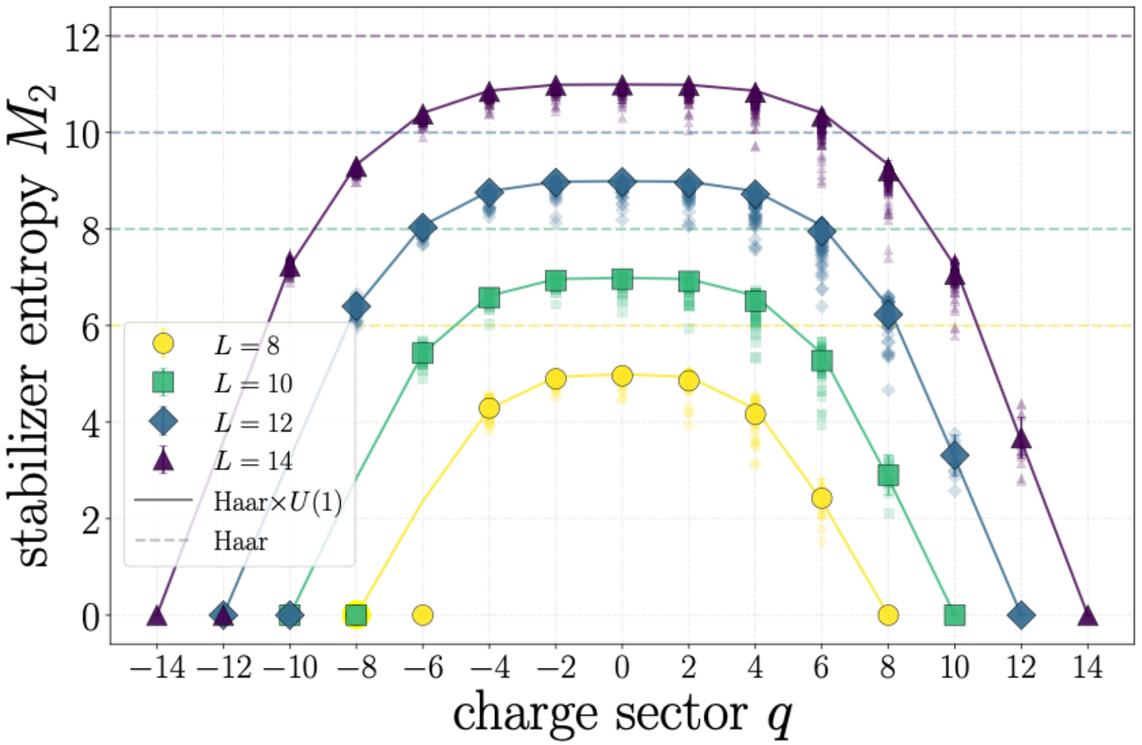
F.J., Dyson. "Statistical theory of the energy levels of complex systems. I." Journal of Mathematical Physics 3.1 (1962): 140-156.

NON-LOCAL INTERACTIONS



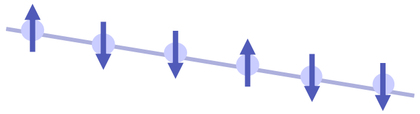
$$Q = 2\sum_{k=1}^{L} c_k^\dagger c_k - \mathrm{L}$$

$$H_{SYK} = \frac{1}{(2L)^{3/2}} \sum_{i,j,k,l=1}^{L} J_{ij;kl}\, c_i^\dagger c_j^\dagger c_k c_l$$



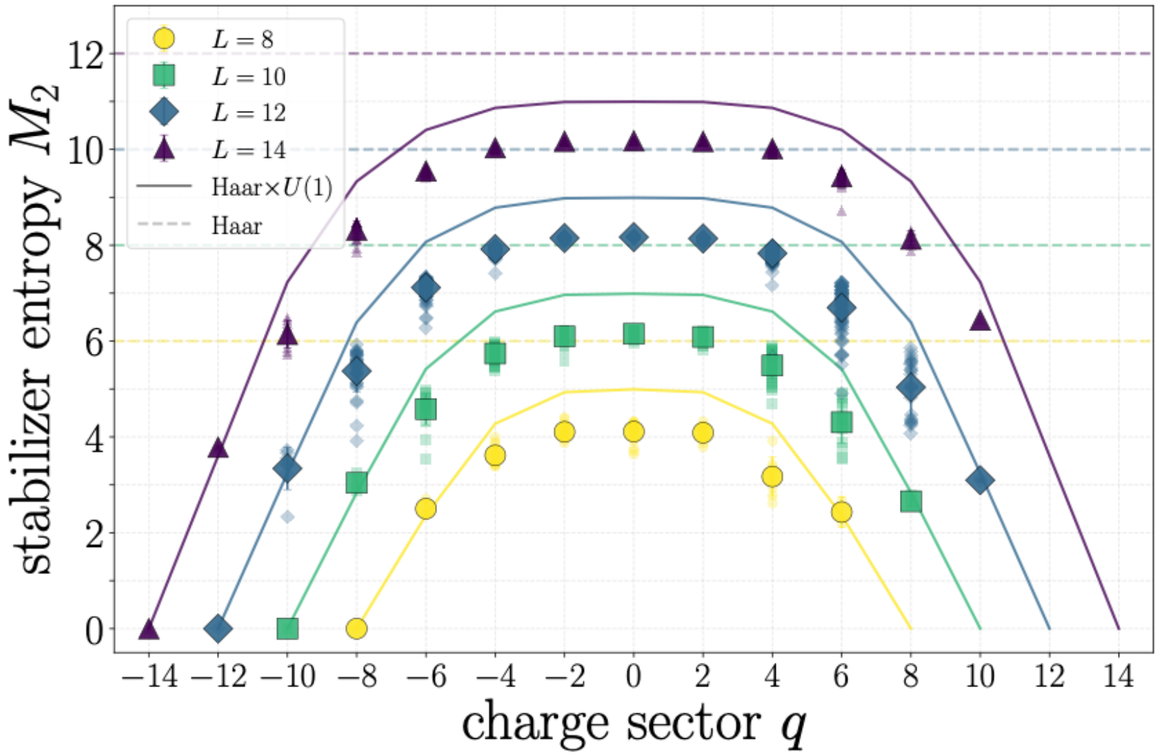
LOCAL INTERACTIONS

$$Q = \sum_{j=1}^{L} Z_j$$

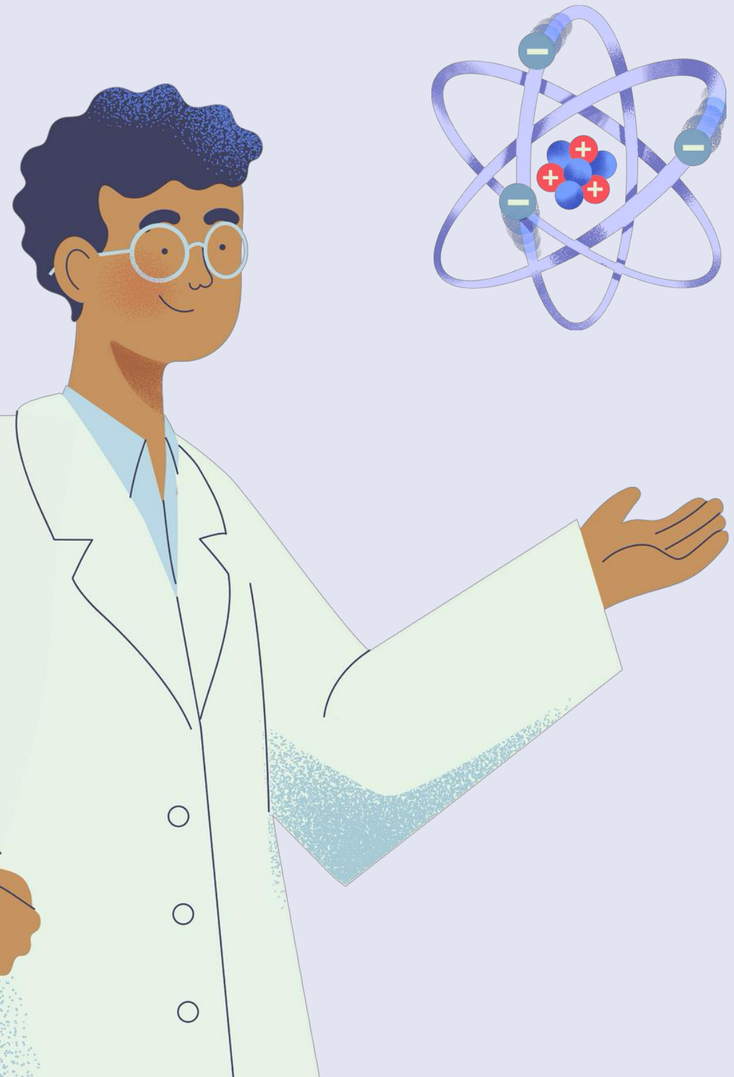


$$H_{\mathrm{MFIM}} = \sum_{j=1}^{L} \mathrm{J}_1(\mathrm{X}_j\mathrm{X}_{j+1} + \mathrm{Y}_j\mathrm{Y}_{j+1}) + \Delta\, Z_j Z_{j+1}$$

$$\mathrm{J}_2\, Z_j Z_{j+1} Z_{j+2} + \mathrm{h_b}(Z_1 - Z_j)$$



D.I. , A. Russotto, B. Jasser, J. Odavic and A. Hamma " Non-stabilizerness and U(1) symmetry in chaotic many-body quantum systems" soon on arxiv



CONCLUSIONS

Entanglement makes room for magic

Magic measures exhibit Van Hove singularities in their PDFs according to the Haar measure

Magic exhibits distinct features for chaotic many-body quantum systems with symmetries, underlying universal constant scaling.